

Rateless Codes for Single-Server Streaming to Diverse Users

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Abstract—We investigate the performance of rateless codes for single-server streaming to diverse users, assuming that diversity in users is present not only because they have different channel conditions, but also because they demand different amounts of information and have different decoding capabilities. The LT encoding scheme is employed. While some users accept output symbols of all degrees and decode using belief propagation, others only collect degree-1 output symbols and run no decoding algorithm. We propose several performance measures, and optimize the performance of the rateless code used at the server through the design of the code degree distribution. Optimization problems are formulated for the asymptotic regime and solved as linear programming problems. Optimized performance shows great improvement in total bandwidth consumption over using the conventional ideal soliton distribution, or simply sending separately encoded streams to different types of user nodes. Simulation experiments confirm the usability of the optimization results obtained for the asymptotic regime as a guideline for finite-length code design.

I. INTRODUCTION

A. Motivation

Growing popularity of ubiquitous computing, along with the surging demand for digital media distribution services such as YouTubeTM, has brought up the issue of efficient media sharing in a heterogeneous network composed of links of diverse quality as well as terminals of varied computing power and demand of media quality.

Consider the air broadcast of digital TV streams. A specialized “plugged” receptor, such as an HDTV set at home, may have more computing power than a small portable device, such as a cellphone, and hence the former might be able to perform more complex decoding algorithms than the latter. Meanwhile, the quality of the broadcast channels may vary due to the location of the receiver, indoors or outdoors, near or far from the transmitting tower. Moreover, devices may need different amounts of data to display a video stream according to screen resolutions.

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Here, we are interested in finding some efficient and yet fair way to provide multicast streaming service to all or a majority of the receivers bearing such heterogeneity. One straightforward solution is to transmit separately encoded data streams suitable for different devices and channels simultaneously, but this requires extra bandwidth and is hence less than efficient.

Rateless codes [1], [2] are, roughly speaking, designed for erasure channels in a way that the set of information symbols may be recovered from any subset of the encoding symbols of size equal or slightly larger than that of the information symbol set by simple decoding. The first practical rateless codes, LT codes, were invented by Michael Luby and published in 2002 [1]. Another class of rateless codes are Raptor codes, a version of which has been written into the 3GPP standard for Multimedia Broadcast/Multicast Service [3].

Rateless codes have the nice features of requiring minimal feedback from the receiver to the sender and operating well over a range of channel conditions. These features are particularly suitable for the broadcast/multicast scenarios. We investigate the possibility of simultaneously serving data sinks of highly heterogeneous decoding capabilities and non-uniform demand of information on channels of diverse quality, with a single rateless coded multicast stream from the source. Specifically, we study the design of LT codes for the multicast streaming purpose.

B. Related Work

The performance of LT codes is determined by the degree distribution of encoding/output symbols. In [1] and [2], the ideal soliton and robust soliton degree distributions have been proposed for minimizing the overhead necessary for recovering all input symbols. However, using these degree distributions when the number of output symbols collected by the receiver is smaller than the total number of the input symbols results in recovery of few input symbols. In [4], the optimal

degree distributions for recovering a constant fraction of the input symbols from the smallest number of output symbols have been studied.

Our work considers multicast streaming to all user nodes with a single data stream. We deal with simultaneous multiple heterogeneities such as link diversity, difference in coding capabilities (e.g., due to limitations in computing resources), and difference volume of information demand (e.g., low or high resolution video). We are interested in performance measures reflecting the collective properties of all the sink nodes of interest, such as maximum and average latency. Our approach by designing

Our paper is organized as follows: Section II introduces the system model for the heterogeneous multicasting network. Section III outlines the guidelines for our optimization problems in the asymptotic regime. Section IV proposes several performance measures and states the corresponding optimization problems. Section V presents the optimization results of the problems formulated in section IV. Section VI contains finite-length simulation results.

II. SYSTEM MODEL: MULTICAST OVER BEC CHANNELS

We consider a streaming network consisting of a single server (source node) and n users (sink nodes) each directly connected to the server by a BEC channel, as shown in Figure 1. The source holds k information

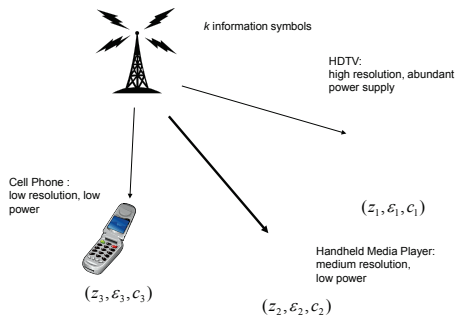


Fig. 1. Broadcast/Multicast System Model

symbols and broadcasts a rateless coded stream to all n sinks. The rateless encoder is an LT encoder[1] with degree distribution with moment generating function

$$P^{(k)}(x) = p_1^{(k)}x + p_2^{(k)}x^2 + \dots + p_k^{(k)}x^k. \quad (1)$$

The LT encoder generates potentially an infinite number of output symbols and broadcast the output stream along all BEC links.

There are two types of sink nodes which differ in the way the LT code is decoded. One type of sinks use the belief propagation (BP) algorithm [1] to recover the input symbols from the received output symbols, while the other type of sinks only accept and recover information from degree-1 output symbols received from the source. The first type are referred to as decoding, and the second as non-decoding sinks. When multiple description [5] encoded, the information symbols allow for tiered reconstruction qualities of the original source information at the sinks.

Sinks are sorted into $1 \leq l \leq n$ clusters, each cluster comprising n_i ($i = 1, 2, \dots, l$) sinks. $n = \sum_{i=1}^l n_i$. A sink in cluster i is characterized by a tuple (z_i, c_i, ϵ_i) ($i = 1, 2, \dots, l$). z_i is a real constant in $[0, 1)$ indicating the fraction of input symbols that sinks in cluster i expect to recover. z_i could be related to the target distortion at the sinks. The two types of sink nodes are distinguished by $c_i = \mathbb{1}_{\{\text{cluster } i \text{ is decoding}\}}$. ϵ_i is the erasure rate of the BEC channels that link the source node to the sink nodes in cluster i . Depending on the performance measure, sinks in the same cluster can often be treated as one single sink because the tuples fully characterize their decoding behavior in this broadcasting scenario.

III. THE OPTIMIZATION PROBLEM IN THE ASYMPTOTIC REGIME

The decoding process of LT codes starts with simply recovering the input symbols connected to the received output symbols of degree-1. This initial recovery induces a new set of output symbols of degree-1. The decoding can continue in the same manner as long as there are output symbols of induced degree-1. Such symbols constitute what is known as the *ripple*. The decoding process halts when the ripple becomes empty. In [2], [6] and [7], the expected size of the ripple throughout of the decoding process is given as a function of the number of unrecovered information symbols. We restate here the part of Theorem 2 in [7] that concerns the expected size of the ripple.

Assume $w \cdot k$ output symbols have been collected and can be used for decoding of an LT code, for some positive constant w . Let $u \cdot k$ be the number of unrecovered information symbols, for a constant $u \in [0, 1)$. Let $r^{(k)}(u)$ be the expected size of the ripple, normalized by k .

Theorem 1: (Maatouk and Shokrollahi [7, Thm. 2]) If an LT code of k information symbols has degree distribution specified by the moment generating function $P^{(k)}(x)$ (see (1)), then

$$r^{(k)}(u) = wu \left(P^{(k)'}(1-u) + \frac{1}{w} \ln u \right) + \mathcal{O}\left(\frac{1}{k}\right), \quad (2)$$

where $P^{(k)'}(x)$ stands for the first derivative of $P^{(k)}(x)$ with respect to x .

The original theorem in [7] is stated for the case where the number of output symbols collected by the receiver is more than the total number of information symbols, i.e., $w > 1$. However, the proof suggests that the theorem also holds for any constant $w < 1$.

Assume that $P^{(k)}(x)$ converges to $P(x) = \sum_{i \geq 1} p_i x^i$ as $k \rightarrow \infty$; then we have

$$r(u) = \lim_{k \rightarrow \infty} r^{(k)}(u) = u(wP'(1-u) + \ln u). \quad (3)$$

In order for the decoding process to carry on until at least a fraction z of the information symbols could be recovered, the ripple size has to be kept positive. If we use the expected value to roughly estimate the ripple size, we should have

$$r(u) = u(wP'(1-u) + \ln u) > 0, \quad \forall u \in (1-z, 1],$$

or equivalently,

$$wP'(1-u) + \ln u > 0, \quad \forall u \in (1-z, 1], \quad (4)$$

Inequality (4) provides a guideline for the design of the degree distribution $P(x)$.

It is interesting to consider the implications of inequality (4) on w and z relationship when the degree distribution is $p_1 = 1$, that is, all output symbols are of degree 1. Then (4) should tell us how many (on the average) output symbols of degree 1 we need in order to recover fraction z of the information symbols. Note that when $p_1 = 1$ we have $P(x) = x$ and $P'(x) = 1$, and in turn from (4), we have $w + \ln u > 0$, $\forall u \in (1-z, 1]$. Thus, $w \geq -\ln(1-z)$, and consequently, the optimal value of w is $-\ln(1-z)$.

Note that we would get the same result if we tried to answer the question about w and z by using the coupon collecting problem, also known as the urns-and-balls problem. Throw a number of balls into k urns. Each ball is thrown independently and falls into each urn with equal probability. What is the number of balls N needed for the number of urns containing at least one ball to reach s ? Note that N is a random variable. It has been derived in [8, Ch. 2] (see also [9]) that the expected number of N is

$$\begin{aligned} \mathbb{E}[N] &= k \left(\frac{1}{k} + \frac{1}{k-1} + \cdots + \frac{1}{k-s+1} \right) \\ &\gtrsim k \ln \frac{k}{k-s+1} = -k \ln \left(1 - \frac{s-1}{k} \right). \end{aligned}$$

Set $z = s/k$, the portion of urns possessing at least one ball. Then, as $k \rightarrow \infty$, $\mathbb{E}[N] \rightarrow -k \ln(1-z)$.

Now, assume that the number of collected output symbols of the LT code specified in Theorem 1 is $W \cdot k$,

where W is a random variable with mean ω , and denote the normalized expected ripple size as $k \rightarrow \infty$ as $r_W(u)$, then

Corollary 2:

$$r_W(u) = u(\omega P'(1-u) + \ln u). \quad (5)$$

Proof: This is due to the linearity of the expected ripple size in W for given u and P .

$$\begin{aligned} r_W(u) &= E \left[Wu \left(P'(1-u) + \frac{1}{W} \ln u \right) \right] \\ &= u \left(E[W] P'(1-u) + \ln u \right) \\ &= u \left(\omega P'(1-u) + \ln u \right) \end{aligned}$$

Then, from 3, we have the recovery condition for random W with mean ω

$$\omega P'(1-u) + \ln u > 0, \quad \forall u \in (1-z, 1], \quad (6)$$

In the next section, we shall use (4) to formulate our optimization problems for LT code degree distribution design.

IV. PERFORMANCE MEASURES AND THEIR OPTIMIZATION PROBLEM STATEMENTS

Recall from Section II tuples (z_i, c_i, ϵ_i) , $i = 1, 2, \dots, l$ are used to characterize the l sink clusters in the streaming network. Let $t_i \cdot k$ be the number of output symbols transmitted by the source up till the time when the sinks in cluster i are able to recover their targeted fraction z_i of the input symbols. Then, the normalized number of symbols a sink in cluster i receives has mean $t_i(1-\epsilon)$.

If cluster i is decoding ($c_i = 1$), then let $x = 1-u$ in (6); we have

$$(1-\epsilon_i)t_i P'(x) + \ln(1-x) > 0, \quad \forall x \in [0, z_i]. \quad (7)$$

A non-decoding user recovering information from a rateless coded stream of degree distribution specified by $P(x)$ is equivalent to a decoding user recovering information from a coded stream of degree distribution specified by $P_0(x) = (1-P'(0)) + P'(0)x$.

If cluster i is non-decoding ($c_i = 0$), then let $p_1 = P'(0)$, the fraction of degree-1 symbols and we have

$$(1-\epsilon_i)t_i p_1 + \ln(1-x) > 0, \quad \forall x \in [0, z_i]. \quad (8)$$

The monotonicity and continuity of the \ln function simplify (8) to

$$(1-\epsilon_i)t_i p_1 + \ln(1-z_i) \geq 0. \quad (9)$$

a) *Min-Max Latency*: In the interest of the transmitting source, we wish to minimize the transmission time that could guarantee the recovery of targeted (z_1, z_2, \dots, z_l) fractions of input symbols by the l sink clusters. In addition, for broadcasting time-sensitive streaming data, new data await to be transmitted after the transmission of an older block of data is finished. Minimizing the maximum latency is especially important for keeping the entire communications scheme in pace.

This optimization problem could be expressed as follows:

$$\begin{aligned} \min_{.P} \quad & \max_i t_i \\ \text{s.t.} \quad & t_i(1 - \epsilon_i)P'(x) + \ln(1 - x) > 0, \quad 0 \leq x \leq z_i, \\ & \quad \text{if cluster } i \text{ is decoding, } i = 1, 2, \dots, l, \\ & t_i(1 - \epsilon_i)p_1 + \ln(1 - z_i) \geq 0, \\ & \quad \text{if cluster } i \text{ is non-decoding, } i = 1, 2, \dots, l, \end{aligned} \quad (10)$$

or equivalently,

$$\begin{aligned} \min_{.P, t_0} \quad & t_0 \\ \text{s.t.} \quad & t_0(1 - \epsilon_i)P'(x) + \ln(1 - x) > 0, \quad 0 \leq x \leq z_i, \\ & \quad \text{if cluster } i \text{ is decoding, } i = 1, 2, \dots, l, \\ & t_0(1 - \epsilon_i)p_1 + \ln(1 - z_i) \geq 0, \\ & \quad \text{if cluster } i \text{ is non-decoding, } i = 1, 2, \dots, l. \end{aligned} \quad (11)$$

Let $t_0^*(z_1, z_2, \dots, z_l)$ be the optimal solution to Problem (11). Then the achievable information recovery region for transmission of $t \cdot k$ output symbols is given by

$$\begin{aligned} Z(t) = \quad & \{(z_1, z_2, \dots, z_l) : \\ & t_0^*(z_1, z_2, \dots, z_l) \leq t, \\ & z_i \in [0, 1], i = 1, 2, \dots, l\}. \end{aligned}$$

As we will see in the next section, optimization results show that, when there are two decoding clusters in the network, one with perfect link conditions and the other with erasure rate $\epsilon = 0.5$, after the source has transmitted $1.6k$ output symbols, the cluster with worse channels can recover 63% of the input symbols in the mean time when the cluster with perfect channels can recover 95%. If the source uses ideal soliton or robust soliton distributions, however, the cluster with worse channels may hardly recover anything until about $2k$ output symbols have been transmitted. Similar results can be seen for cases where there is one decoding cluster and a non-decoding cluster present in the network.

b) *Max-Min Channel Utilization*: The Shannon capacity of the BEC link to sink cluster i is $(1 - \epsilon_i)$ bits per channel use. The channel utilization of a link to cluster i is then $v_i = \frac{z_i}{(1 - \epsilon_i)t_i}$. We wish to maximize the minimum

channel utilization on all links, which is equivalent to minimizing the inverse of the channel utilization.

$$\begin{aligned} \min_{.P} \quad & \max_i \frac{t_i(1 - \epsilon_i)}{z_i} \\ \text{s.t.} \quad & t_i(1 - \epsilon_i)P'(x) + \ln(1 - x) \geq 0, \quad 0 \leq x \leq z_i, \\ & \quad \text{if cluster } i \text{ is decoding, } i = 1, 2, \dots, l, \\ & t_i(1 - \epsilon_i)p_1 + \ln(1 - z_i) \geq 0, \\ & \quad \text{if cluster } i \text{ is non-decoding, } i = 1, 2, \dots, l, \end{aligned} \quad (12)$$

or equivalently,

$$\begin{aligned} \min_{.P, v_0} \quad & v_0 \\ \text{s.t.} \quad & v_0 z_i P'(x) + \ln(1 - x) \geq 0, \quad 0 \leq x \leq z_i, \\ & \quad \text{if cluster } i \text{ is decoding, } i = 1, 2, \dots, l, \\ & v_0 z_i p_1 + \ln(1 - z_i) \geq 0, \\ & \quad \text{if cluster } i \text{ is non-decoding, } i = 1, 2, \dots, l. \end{aligned} \quad (13)$$

Maximizing the min channel utilization proves to be irrelevant to the channel conditions, as may be inferred from the expression of Problem (13). As we will see in the next section, high minimum channel utilization could be achieved when the decoding cluster has either a very low or a very high demand. The increase in the demand of the non-decoding cluster, on the other hand, always degrades channel utilization.

c) *Max-Min Throughput*: The throughput at each sink cluster i may be defined as $\frac{z_i}{t_i}$. It is of interest to measure the objective channel degradation regardless of channel capacity so as to provide reference for service pricing of the broadcast application. We wish to maximize the minimum throughput of all sink clusters. This is equivalent to minimizing the maximum of the inverse of the throughput. The optimization problem is therefore expressed as Problem (14):

$$\begin{aligned} \min_{.P} \quad & \max_i \frac{t_i}{z_i} \\ \text{s.t.} \quad & t_i(1 - \epsilon_i)P'(x) + \ln(1 - x) \geq 0, \quad 0 \leq x \leq z_i, \\ & \quad \text{if cluster } i \text{ is decoding, } i = 1, 2, \dots, l, \\ & t_i(1 - \epsilon_i)p_1 + \ln(1 - z_i) \geq 0, \\ & \quad \text{if cluster } i \text{ is non-decoding, } i = 1, 2, \dots, l, \end{aligned} \quad (14)$$

or equivalently,

$$\begin{aligned} \min_{.P, w_0} \quad & w_0 \\ \text{s.t.} \quad & w_0 z_i (1 - \epsilon_i)P'(x) + \ln(1 - x) \geq 0, \quad 0 \leq x \leq z_i, \\ & \quad \text{if cluster } i \text{ is decoding, } i = 1, 2, \dots, l, \\ & w_0 z_i (1 - \epsilon_i)p_1 + \ln(1 - z_i) \geq 0, \\ & \quad \text{if cluster } i \text{ is non-decoding, } i = 1, 2, \dots, l. \end{aligned} \quad (15)$$

d) *Minimum Average Latency*: We are also interested in minimizing the average latency of all sinks. This is a natural measure of overall performance.

$$\begin{aligned} \min_{P, n} \quad & \frac{\sum_{i=1}^l n_i t_i}{n} \\ \text{s.t.} \quad & t_i(1 - \epsilon_i)P'(x) + \ln(1 - x) \geq 0, 0 \leq x \leq z_i, \\ & \text{if cluster } i \text{ is decoding, } i = 1, 2, \dots, l, \\ & t_i(1 - \epsilon_i)p_1 + \ln(1 - z_i) \geq 0, \\ & \text{if cluster } i \text{ is non-decoding, } i = 1, 2, \dots, l. \end{aligned} \quad (16)$$

Optimization results show that, when all channels are perfect and half of the sinks are decoding, half non-decoding, the optimized achievable average latency with one single broadcast data stream is mostly worse than broadcasting on separate channels data streams individually optimized for different sinks. Details are presented in the Section V.

Since our objectives are the minimization of increasing functions of the latencies, with arguments similar to Lemma 2 of [4], we can claim that there must exist optimal solutions to Problems (11), (13), (15) and (16) with polynomials $P(x)$ of degree no higher than $d_{\max} = \lceil \frac{1}{1 - \max_i \{z_i\}} \rceil - 1$. This promises the ready conversion of Problems (11), (13) and (15) into linear programming problems by the method proposed in [4]. Problem (16) may be converted to a series of linear programming problems for fixed $p_1 \in [0, 1]$ when there are only two sink clusters in the network, one decoding and the other non-decoding. To solve the linear programming problems numerically, the parameter x in the constraints is evaluated at discrete points and lower bounds for the minimization problems with constraints continuous in x are obtained. In the next section we will show in detail the optimization results of these problems.

V. OPTIMIZATION RESULTS

A. Application to 2-Cluster Situations

Now we apply our optimization problem to the case where only two sink clusters with distinct tuple characteristics, (z_1, c_1, ϵ_1) and (z_2, c_2, ϵ_2) exist. We deal with: (1) $c_1 = c_2 = 1$, $\epsilon_1 = 0, \epsilon_2 = 0.5$: both clusters are decoding, but with diverse channel conditions; (2) one cluster is decoding while the other is not, with equal or diverse channel qualities.

Figure 2 shows the contour graphs of the outer bounds of the min-max latency on the $z_1 - z_2$ plane for four typical cases.

- Dense contour regions indicate the regions where the minimized maximum latency is sensitive in z_1 or z_2 ;

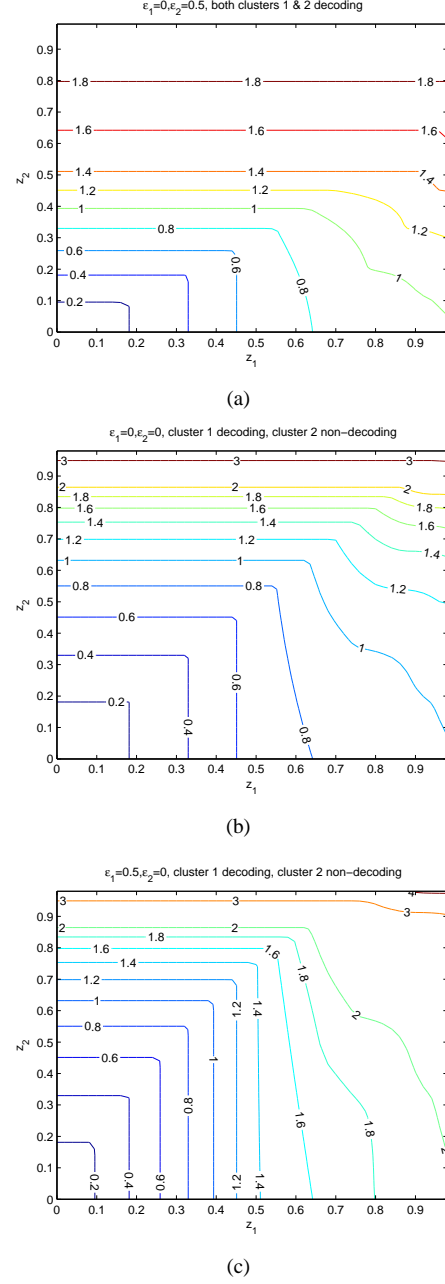


Fig. 2. Contour graphs of the numerical lower bounds of the min-max latency. Contours define the outer bounds of achievable (z_1, z_2) regions given a specific number of transmitted output symbols. Drawn from the solution to Problem (11).

- Vertical(or horizontal) contour sections indicate regions where z_1 (or z_2) is the bottleneck of latency;
- Steep(or gradual) contour sections indicate z_1 (or z_2)-dominant regions: reducing z_1 (or z_2) a bit trades for a bigger advance in z_2 (or z_1) for fixed min-max latency. These are the regions where the degree distribution of the LT encoder could be finely tuned for the two clusters to finish reception at the same time.

Figures 3(a) and 3(b) show respectively the contour graphs of the outer bounds of the max-min channel utilization when both sink clusters are decoding and when one cluster is decoding but the other is not.

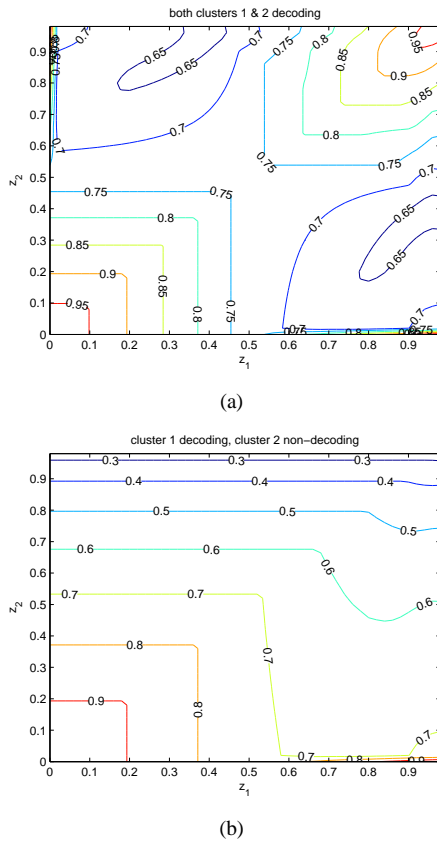


Fig. 3. Contour graphs of the achievable max-min channel utilization. Drawn from the solution to Problem (12).

- The results are irrelevant to the channel quality;
- Both clusters are decoding (Figure 3(a)):
 - For uniform demand $z = z_1 = z_2$, channel utilization is the same as the slope of the outbound curve in the $z - r$ plot in [4]: lowest as z approaches 0.5 and highest when z is near 0 or 1.

- For non-uniform demand however diverse, the max-min channel utilization is better than 64%;
- Cluster 1 is decoding while cluster 2 is not(Figure 3(b)):
 - Max-min channel utilization decreases with increasing z_2 ;
 - The “lowest in the middle” phenomenon could still be observed when z_2 is small;
 - The minimum channel utilization could drop below 40%.

For the results of maximizing the minimum throughput, we choose to show the outbounds of the optimal solutions for $z_1 = z_2$ under different channel and decoding conditions in Figure 4.

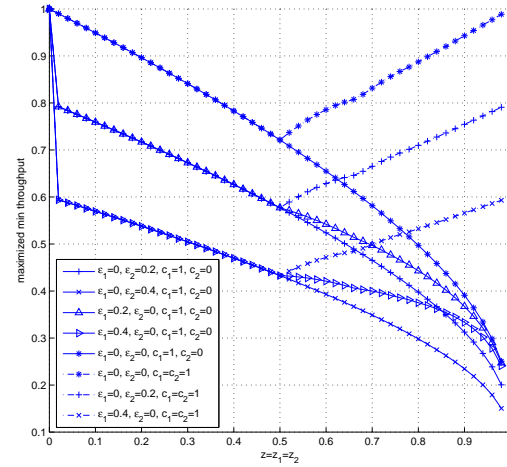


Fig. 4. Max-min throughput versus $z = z_1 = z_2$ under various channel conditions. Drawn from the solution to Problem (14).

As shown in Figure 4

- The max-min throughput cannot go over the capacity of the worse channel, as expected;
- The curves for both clusters decoding in different channel conditions are almost parallel and similar to the trend of the channel utilization, which is also expected because of the uniform demand assumed here;
- The curves for cluster 1 decoding and cluster 2 non-decoding is always dropping with the growth of z ; however, when the demand is not uniform, when z_2 is small enough and z_1 large enough, an increase in throughput could still be observed;
- The distance between the outerbound max-min throughput curves for one cluster decoding and the other not becomes smaller as $z = z_1 = z_2$

grows larger, which implies the less sensitivity of the optimized minimum throughput to channel conditions when z is larger.

Figure 5 shows the solution to Problem (16), minimizing the average latency.

- As shown in Figure 5(a), on a perfect channel, even when half of the output symbols are of degree-1, a decoding sink may be able to decode 99% of all the information symbols with an overhead of less than 16% of the size of the set of information symbols;
- As shown in 5(c), as the portion of decoding cluster increases from 0 to 1, the fraction of degree-1 output symbols in the optimized degree distribution gracefully decreases from 1 to 0.

B. Comparison of Performance

Table I lists a comparison of the total number of transmitted symbols to fulfill the demands of two clusters under four streaming schemes:

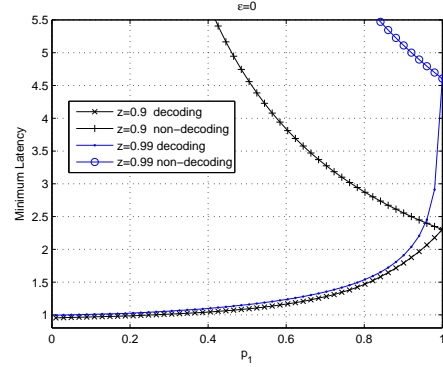
- Scheme A0: The source sends a single stream to all sinks, minimizing max latency.
- Scheme A1: The source sends a single stream to all sinks, minimizing latency of cluster 1.
- Scheme A2: The source sends a single stream to all sinks, minimizing latency of cluster 2.
- Scheme A12: The source sends two independent streams to the clusters, each minimizing latency of the targeted cluster.

TABLE I
COMPARISON OF TOTAL NUMBER OF TRANSMITTED SYMBOLS
UNDER FOUR STREAMING SCHEMES

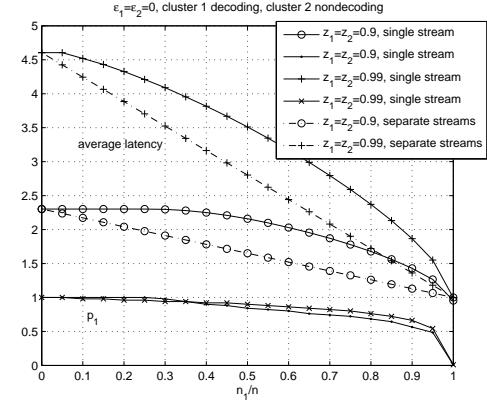
	(z_1, c_1, ϵ_1) (z_2, c_2, ϵ_2)	Scheme A0		Scheme A1	
		either	total	either	total
cluster 1	(0.98, 1, 0)	1.5634	1.5634	0.9914	∞
cluster 2	(0.72, 0, 0)	1.5634		∞	
cluster 1	(0.98, 1, 0)	1.6220	1.6220	0.9914	1.9828
cluster 2	(0.63, 1, 0.5)	1.6220		1.9828	
	(z_1, c_1, ϵ_1)	Scheme A2		Scheme A12	
		either	total	either	total
cluster 1	(0.98, 1, 0)	3.9120	3.9120	0.9914	2.2644
cluster 2	(0.72, 0, 0)	1.2730		1.2730	
cluster 1	(0.98, 1, 0)	1.9959	1.9959	0.9914	2.5696
cluster 2	(0.63, 1, 0.5)	1.5782		1.5782	

Scheme A0 performs significantly better than Schemes A1, A2 and A12 in terms of the total number of output symbols transmitted by the source.

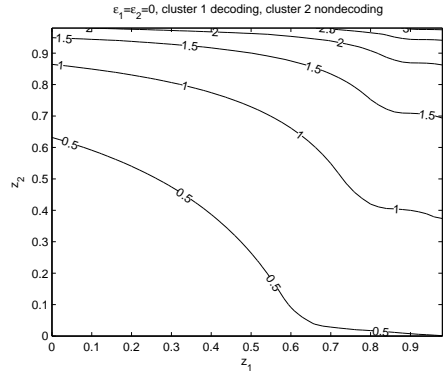
When considering average latency for multicasting to both decoding and non-decoding clusters on perfect channels, however, it could be seen from Figure 5(b) that transmitting separately encoded streams on separate channels (Scheme A12) is better than transmitting a single stream (Scheme A0).



(a) Minimum achievable latency of the decoding cluster vs. p_1 over perfect channel



(b) Minimum average latency and achieving p_1 versus size of decoding cluster, $z_1 = z_2$.



(c) Minimum average latency contour graph for half-half decoding-non-decoding

Fig. 5. Results for minimizing average latency, Problem (16).

VI. FINITE-LENGTH SIMULATION

Figure 6(a) gives the simulated sample curves of information recovery versus latency when the decoding cluster targets at recovering 80% of the input symbols and the non-decoding cluster targets at recovering 40%. The distribution of the latency till the two clusters achieve targeted information recovery is given in 6(b). The empirical average value of t_0 is 1.0718, 2.3% greater than the optimization result $t_0^* = 1.0473$, which is in acceptable error range.

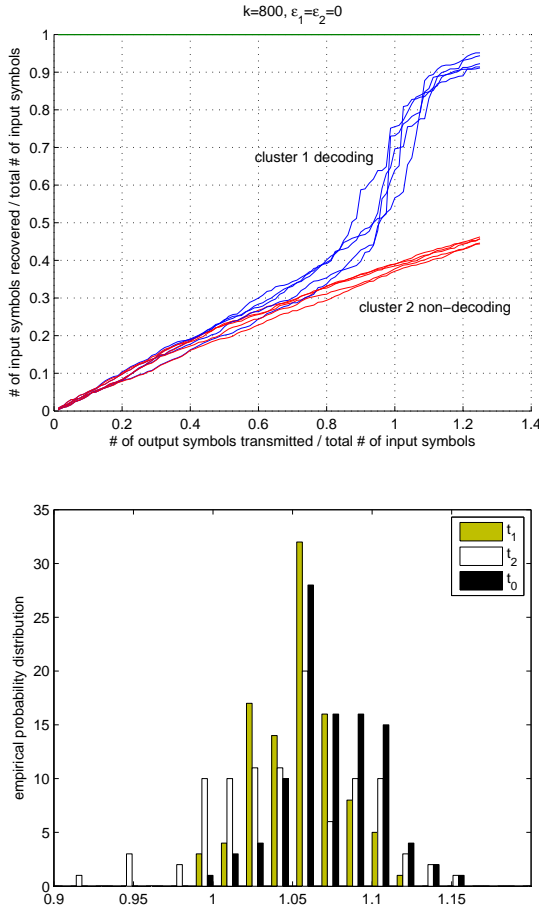


Fig. 6. (a) Finite-length simulated time progress of information recovery for degree distribution $P(x) = 0.4878x + 0.4878x^4 + 0.0244x^5$, optimized for min-max latency and $z_1 = 0.8$, $z_2 = 0.4$, $\epsilon_1 = \epsilon_2 = 0$, the number of information symbols being $k = 800$. 5 simulation instances plotted. (b) Empirical probability distribution of latency t_1 and t_2 , obtained from 100 samples; mean of t_1 is 1.0532, standard deviation 0.0263; mean of t_2 is 1.0451, standard deviation 0.0443; mean of $t_0 = \max\{t_1, t_2\}$ is 1.0718, standard deviation 0.0300. Optimization results give that for $(z_1, z_2) = (0.8, 0.4)$, min-max latency is $t_0^* = 1.0473$.

VII. CONCLUDING REMARKS

In this work, we have investigated the performance of LT rateless codes for streaming from a single server to diverse users. The degree distributions of the LT-output symbols have been optimized according to network parameters. The degree distribution optimization problems have been formulated in the asymptotic regime and solved numerically, and simulations have been conducted to confirm the usability of the asymptotic results as a guideline for finite-length code design. The impact of diversity in channel conditions, non-uniform demands and coding methods of users on transmission latency, channel utilization and throughput have also been shown through the optimization results. As demonstrated in Section V, following our scheme, the total bandwidth consumption for satisfying diverse users is considerably reduced compared to either sending separate streams for different users or sending a stream that is optimized for only one of the users.

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